Our calculations and considerations show that a simple experiment on uniaxial compression turns out to be rather complicated from the point of view of analysis. The stressed state of a sample is essentially nonuniform, and the fracture conditions are satisfied first in the nonuniform region of the pattern near the pressure plates of the testing machine. As a result the uniaxial compressive strength is a convenient technical strength characteristic of a structural sample rather than a characteristic of the material.

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CONVECTIVE EFFECTS IN LIQUID INCLUSIONS DRIFTING IN NONUNIFORMLY HEATED SOLIDS

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§1. We will consider a liquid-filled spherical cavity in an infinite solid mass. The liquid dissolves the surrounding material and under equilibrium conditions is a saturated solution of concentration C_0. At infinity let there be a constant horizontal temperature gradient $\nabla T_e = \mathbf{A}$. Under these conditions in the gravitational field $\mathbf{g}$ free convective motion develops in the liquid.

We assume that the motion is slow and steady; solid phase can crystallize out of the supersaturated solution only at the interface between the inclusion and the matrix; the dissolving of the solid in the liquid does not lead to a change in the volume of the latter; the thermal diffusion and diffusion heat-conduction effects are negligible [1]. All the parameters (kinematic and dynamic viscosity coefficients ν and η, thermal conductivity $\varkappa$, thermal diffusivity χ, and diffusion coefficient D) of the liquid and the solid are constant. The solubility C_0 and the liquid density ρ depend linearly on temperature T. We assume that the density also depends on the concentration C, defined as the ratio of the mass of solid material per unit volume of solution to the mass of that volume:

$$\rho(T, C) = \rho(T_0, C_0)[1 + \alpha(C - C_0) - \beta(T - T_0)],$$

$$C_0(T) = C_0(T_0) + (dC_0/dT)(T - T_0).$$

The nonuniform heating of the walls of the cavity leads to the dissolving of the hotter parts of the solid and subsequent diffusive and convective mass transfer to the cooler regions, where the solution is supersaturated

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and part of the matrix material is precipitated on the cavity walls. The inclusion begins to migrate through the solid. The drift velocity **u** and the change in the shape of the inclusion must be determined in the course of obtaining the solution.

The motion of liquid inclusions in crystals in the presence of a temperature gradient was studied quantitatively in [1-3]. The theoretical estimates of the drift velocity given in those studies do not take convective effects into account.

Let the motion already be steady at the initial instant t = 0. Then in the reference system associated with the solid, the distribution of velocities **v**, pressures p, temperatures T, and concentrations C in the liquid and temperatures T_e in the solid is given by the convection equations in the Boussinesq approximation:

$$\partial \mathbf{v}/\partial t + (\mathbf{v}\nabla)\mathbf{v} = -\nabla p/\rho + \nu\Delta\mathbf{v} + \mathbf{g}\alpha(C - C_0) - \mathbf{g}\beta(T - T_0),$$
$$\operatorname{div}\mathbf{v} = 0,\ \partial C/\partial t + \mathbf{v}\nabla C = D\Delta C, \tag{1.1}$$
$$\partial T/\partial t + \mathbf{v}\nabla T = \chi\Delta T,\ \partial T_e/\partial t = \chi_e\Delta T_e,$$

here and in what follows the subscript "e" denotes that the quantity relates to the solid; functions without a subscript relate to the liquid; **g** is the acceleration of gravity.

To (1.1) and the conditions at infinity formulated above it is necessary to add the boundary conditions at the surface of the inclusion. Since, by assumption, the volume of the droplet does not change, the velocity of the liquid at the interface with the solid is equal to the drift velocity **u**. The usual conditions of equality of temperatures and heat and mass fluxes are also imposed. The concentration is equal to the solubility C_0 at the corresponding surface temperature.

We now go over to the reference system associated with the drifting cavity. We direct the polar axis z of the spherical coordinate system (r, ϑ, φ) upward, and make the coordinate origin coincide with the center of mass of the droplet. The angle φ is reckoned from the x axis of the Cartesian coordinate system (x, y, z), the direction of the unit vectors **i**, **j**, **k** being determined by the direction of the temperature gradient $\mathbf{A} = A\mathbf{j}$ and the acceleration of gravity $\mathbf{g} = -g\mathbf{k}$. The temperature and concentration are reckoned from the undisturbed values of the functions at the point at which the center of mass of the droplet is located at the time t in question. Then the new "primed" functions are related with the old ones as follows:

$$\mathbf{v} = \mathbf{v}' + \mathbf{u},\ T = \mathbf{A}\mathbf{u}t + T',$$
$$T_e = \mathbf{A}\mathbf{u}t + T_e',\quad C = C_0\left(1 + \frac{dC_0}{dT}\mathbf{A}\mathbf{u}t\right) + C'. \tag{1.2}$$

We now go over to dimensionless quantities, for which purpose we select as units of length, velocity, temperature, concentration, pressure, and time the mean radius of the inclusion a, $g\beta a^3 A/\nu$, aA, $\beta a A/\alpha$, $g\beta a^3 A\rho$, a^2/ν, respectively. Denoting the dimensionless variables by the same letters but without primes, using (1.1) and (1.2) we obtain equations for the dimensionless quantities in the reference system associated with the center of mass of the droplet:

$$\partial \mathbf{v}/\partial t + \mathrm{Gr}(\mathbf{v}\nabla)\mathbf{v} = -\nabla p + \Delta\mathbf{v} + (T - C)\mathbf{k},$$
$$\mathrm{Sc}\,\partial C/\partial t + \mathrm{Sc}\mathrm{Gr}(K\mathbf{u}\mathbf{j} + \mathbf{v}\nabla C) = \Delta C,\ \operatorname{div}\mathbf{v} = 0, \tag{1.3}$$
$$\mathrm{Pr}\,\partial T/\partial t + \mathrm{Pr}\mathrm{Gr}(\mathbf{u}\mathbf{j} + \mathbf{v}\nabla T) = \Delta T,$$
$$\sigma\,\partial T_e/\partial t + \sigma\mathrm{Gr}(\mathbf{u}\mathbf{j} - \mathbf{u}\nabla T_e) = \Delta T_e,$$

where $\mathrm{Pr} = \nu/\chi$ and $\sigma = \nu/\chi_e$ are the Prandtl numbers; $\mathrm{Sc} = \nu/D$ is the Schmidt number. The liquid parameter $K = \alpha(dC_0/dT)/\beta$ characterizes the relation between solubility and temperature. $\mathrm{Gr} = g\beta a^4 A/\nu^2$ is the Grashof number.

The presence in the left sides of (1.3) of terms proportional to the drift velocity **u** is associated with the choice of reference temperature and concentration. Allowing for these terms leads to the growth of the inclusion. Since experiments have shown [4] that the volume of the inclusion remains constant, these terms can be neglected.

To (1.3) we add the boundary conditions at infinity $T_e = r\sin\vartheta\sin\varphi$ and at the surface of the cavity $r = R(\vartheta,\ \varphi)$

$$\mathbf{v} = 0,\ T = T_e,\ C = KT,$$
$$\partial T/\partial n = \varkappa\partial T_e/\partial n = \gamma\mathbf{u}\mathbf{n},\ \partial C/\partial n = \rho\mathbf{u}\mathbf{n}, \tag{1.4}$$

where $\mathbf{n}$ is the unit vector of the surface normal $R(\vartheta, \varphi)$; $\gamma = \Delta H g\beta a^3\rho_e/\nu\varkappa$ is the dimensionless specific heat of solution ΔH ($\gamma > 0$ corresponds to the release of heat); the parameter $\rho = \rho_e g a^3 \alpha/\rho D\nu$ relates the diffusion mass transfer at the boundary with the drift velocity; $\varkappa = \varkappa_e/\varkappa$ is the thermal-conductivity ratio.

We will seek the stationary ($\partial/\partial t = 0$) solution of problem (1.3), (1.4) in the form of an expansion in powers of the small parameter Gr [5] (in experiments [2] on the migration of water "droplets" in KCl crystals the Grashof number was of the order of 10^{-6})

$$\begin{gathered} T = T_0 + \mathrm{Gr}\,T_1 + \ldots,\ T_e = \Theta_0 + \mathrm{Gr}\,\Theta_1 + \ldots, \\ C = C_0 + \mathrm{Gr}\,C_1 + \ldots,\ \mathbf{u} = \mathbf{u}_0 + \mathrm{Gr}\,\mathbf{u}_1 + \ldots, \\ \mathbf{v} = \mathbf{v}_0 + \ldots,\ R(\vartheta, \varphi) = 1 + \mathrm{Gr}\,h_1(\vartheta, \varphi) + \ldots \end{gathered} \tag{1.5}$$

Substituting (1.5) in (1.3), (1.4), we obtain

$$\begin{gathered} \mathbf{v}_0 = [a(1-K)/20](r^3 - r)\mathbf{r}\times\nabla(\sin\vartheta\cdot\cos\varphi), \\ T_0 = ar\sin\vartheta\cdot\sin\varphi,\ \Theta_0 = (r + b/r^2)\sin\vartheta\cdot\sin\varphi, \\ C_0 = Kar\sin\vartheta\cdot\sin\varphi;\ \mathbf{u}_0 = (aK/\rho)\mathbf{j},\ a = 3\varkappa\psi, \\ b = (\varkappa - 1 + \gamma K/\rho)\psi,\ \psi^{-1} = 2\varkappa + 1 - \gamma K/\rho, \\ T_1 = (er + (p/28)r^5 - (p/10)r^3)\cos\vartheta,\ p = \mathrm{Pr}\,a^2(1-K)/20, \\ C_1 = (dr + (s/28)r^5 - (s/10)r^3)\cos\vartheta,\ s = \mathrm{Sc}\,Ka^2(1-K)/20, \\ e = (\psi/140)[17p + 9p(2\varkappa - K\gamma/\rho) - 8s\gamma/\rho], \\ d = (\psi/140)[8Kp + 9p(2\varkappa + 1) - 17sK\gamma/\rho], \\ \Theta_1 = \left(p - \frac{s\gamma}{\rho}\right)\frac{2\psi}{35}\frac{\cos\vartheta}{r^2},\quad \mathbf{u}_1 = [Kp - s(2\varkappa + 1)]\frac{2\psi}{35\rho}\mathbf{k}. \end{gathered} \tag{1.6}$$

The function $h_1(\vartheta, \varphi)$ in the expansion of $R(\vartheta, \varphi)$ is also determined in the course of obtaining the solution and proves to be equal to zero.

§2. We will consider problem (1.3), (1.4) with the modified condition at infinity $T_e = -r\cos\vartheta$ (heating from below, $\mathbf{A} = -A\mathbf{k}$). We will seek the stationary ($\partial/\partial t = 0$) solution of system of equations (1.3), discarding, as in Sec. 1, the terms proportional to $\mathbf{u}$ in the left sides of the equations. Using boundary conditions (1.4) and the new condition at infinity, we obtain

$$\begin{gathered} \mathbf{v} = 0,\ T = -ar\cos\vartheta,\ C = -Kar\cos\vartheta, \\ T_e = (-r - b/r^2)\cos\vartheta,\ \mathbf{u} = |\mathbf{u}_0|\mathbf{k}. \end{gathered} \tag{2.1}$$

The constants of integration a, b, and $\mathbf{u}_0$ were written out in (1.6).

Diffusion solution (2.1) will be unstable at a certain Gr_* [the Grashof number can be both positive (heating from below) and negative (heating from above)]. To determine the critical Grashof number Gr_* we use the standard procedure of [5]. Linearizing Eqs. (1.3) with respect to small perturbations of velocity $\mathbf{v}$, pressure p, temperatures τ and τ_e, and concentration c, and assuming that all the quantities depend on time t in accordance with the law $\exp(-\lambda t)$, we obtain

$$\begin{gathered} -\lambda\mathbf{v} = -\nabla p + \Delta\mathbf{v} + (\tau - c)\mathbf{k}, \\ \operatorname{div}\mathbf{v} = 0,\ \lambda\sigma\tau_e - \Delta\tau_e, \\ -\lambda\,\mathrm{Pr}\,\tau + \mathrm{Pr}\,\mathrm{Gr}\,\mathbf{v}\nabla T = \Delta\tau, \\ -\lambda\,\mathrm{Sc}\,c + \mathrm{Sc}\,\mathrm{Gr}\,\mathbf{v}\nabla C = \Delta c. \end{gathered} \tag{2.2}$$

To (2.2) we add the boundary conditions at the surface of the inclusion $r = 1$,

$$\mathbf{v} = 0,\quad \tau = \tau_e,\quad \frac{\partial\tau}{\partial r} - \varkappa\frac{\partial\tau_e}{\partial r} = \frac{\gamma}{\rho}\frac{\partial c}{\partial r},\quad c = K\tau \tag{2.3}$$

and at infinity $\tau_e = 0$.

We seek the solutions of Eqs. (2.2) in the form

$$\begin{gathered} \mathbf{v} \sim f(r)\mathbf{r}\times\nabla(\sin\vartheta\cdot\cos\varphi),\ p = q(r,\vartheta)\sin\varphi, \\ c \sim \tau \sim f(r)\sin\vartheta\cdot\sin\varphi,\ f(r) = \frac{\sin kr}{(kr)^2} - \frac{\cos kr}{kr}. \end{gathered} \tag{2.4}$$

Substituting (2.4) in (2.2) and eliminating, as in [6], the pressure, we obtain a homogeneous algebraic system of equations for the perturbation amplitudes. Equating the determinant of this system to zero, we find the equation for the characteristic decrements λ:

$$\lambda^3 \Pr \mathrm{Sc} - \lambda^2 k^2 (\Pr + \mathrm{Sc} + \Pr \mathrm{Sc}) + \lambda k^4 \left[1 + \Pr + \mathrm{Sc} + \frac{\mathrm{Ra}\,\mathrm{Sc}\,(K-1)}{2k^4}\right] - k^2 \left[k^4 - \frac{\mathrm{Ra}}{2}\left(1 - \frac{K\,\mathrm{Sc}}{\Pr}\right)\right] = 0, \tag{2.5}$$

where $\mathrm{Ra} = a \Pr \mathrm{Gr}$ is the Rayleigh number.

The decrement λ may be complex $\lambda = \delta + i\omega$; δ indicates the decay ($\delta > 0$) or growth ($\delta < 0$) of the perturbation; ω defines their frequency. The neutral line $\delta = 0$ corresponds to the boundary of monotonic ($\omega = 0$) or oscillatory ($\omega \neq 0$) instability.

We will determine the boundary of monotonic instability, substituting $\lambda = 0$ in (2.5):

$$k_1 = 0,\ k_{2,3} = \pm k,\ 2k^4 = \mathrm{Ra}_*(1 - K\,\mathrm{Sc}/\Pr). \tag{2.6}$$

Using (2.4), (2.6) we obtain the general solution of (2.2) satisfying the condition at infinity:

$$\begin{gathered} \mathbf{v} = [c_1 f(r) + c_2 g(r)]\mathbf{r} \cdot \nabla(\sin\vartheta \times \cos\varphi), \\ \tau = [-(\mathrm{Ra}_*/k^2)(c_1 f - c_2 g) + c_3 r]\sin\vartheta \cdot \sin\varphi,\ \tau_e = c_4/r^2, \\ c = [-(\mathrm{Ra}_* K\,\mathrm{Sc}/k^2 \Pr)(c_1 f - c_2 g) + c_3 r]\sin\vartheta \cdot \sin\varphi, \\ g(r) = \frac{\mathrm{sh}\,kr}{(kr)^2} - \frac{\mathrm{ch}\,kr}{kr}. \end{gathered} \tag{2.7}$$

Substituting (2.7) in boundary conditions (2.3) and equating to zero the determinant formed from the coefficients of c_i ($i = 1, \ldots, 4$), we obtain

$$\frac{K(\Pr - \mathrm{Sc})\,\rho\left(1 + 2\varkappa - \frac{\gamma}{\rho}\right)}{2(K-1)(\rho \Pr - \gamma K\,\mathrm{Sc})} = \varkappa + \left[\frac{k^3(\mathrm{ctg}\,k - \mathrm{cth}\,k)}{4(1 - k\,\mathrm{cth}\,k)(1 - k\,\mathrm{ctg}\,k)} - 1\right]. \tag{2.8}$$

Equation (2.8), together with the equation for k in (2.6), determines the boundary of monotonic instability Ra_*.

As $\varkappa \to \infty$ the equation for Ra_* takes the form

$$\mathrm{Ra}_*(\varkappa \to \infty) = 815/(1 - K\,\mathrm{Sc}/\Pr). \tag{2.9}$$

It is known [6] that instability of the diffusion heat transfer through a spherical cavity with a fixed surface temperature develops at the critical Rayleigh number $\mathrm{Ra}_* = 815$. The change by a factor $(1 - K\mathrm{Sc}/\Pr)$ in the present problem can be explained as follows.

Let us consider the virtual displacement of a volume ΔV in a liquid heated from below. An upward displacement brings the element ΔV into a region whose temperature is ΔT lower and where the concentration of the heavy component is $\Delta C \sim K\Delta T$ less. Diffusion solution (2.1) is unstable if excess heavy component diffuses from the volume ΔV (characteristic time $\tau_g \sim \Delta C/D \sim K\Delta T/D$) but the volume is unable to cool (characteristic cooling time $\tau_T \sim \Delta T/\chi$). Heating from below is unstable if $\tau_g/\tau_T \approx K\mathrm{Sc}/\Pr < 1$ and the critical Rayleigh number [cf. (2.9)] differs from 815 by a factor $(1 - K\mathrm{Sc}/\Pr)$.

In order to determine the boundary of oscillatory instability, we substitute $\lambda = i\omega$ in (2.5) and eliminate

$$\begin{gathered} k_1 = 0,\ k_2 = k,\ k_3 = ik, \\ 2k^4 \frac{(\Pr + \mathrm{Sc})(1 + \mathrm{Sc})}{\mathrm{Sc}^2} = \mathrm{Ra}\left(1 - \frac{1 + \mathrm{Sc}}{1 + \Pr}\frac{K\Pr}{\mathrm{Sc}}\right). \end{gathered} \tag{2.10}$$

Using (2.10) and (2.4), we write the general solution of system (2.2), satisfying the condition $\tau_e = 0$ at infinity, in the form

$$\begin{gathered} \mathbf{v} = (c_1 f + c_2 g + c_3 r)\mathbf{r} \times \nabla(\sin\vartheta \cdot \cos\varphi), \\ \tau = \mathrm{Ra}\left(\frac{c_1 f}{i\omega\Pr - k^2} + \frac{c_2 g}{i\omega\Pr + k^2} + \frac{c_3 r}{i\omega\Pr}\right)\sin\vartheta \cdot \sin\varphi, \\ c = \mathrm{Ra}\frac{K\,\mathrm{Sc}}{\Pr}\left(\frac{c_1 f}{i\omega\mathrm{Sc} - k^2} + \frac{c_2 g}{i\omega\mathrm{Sc} + k^2} + \frac{c_3 r}{i\omega\mathrm{Sc}}\right)\sin\vartheta \cdot \sin\varphi, \\ \tau_e = c_4 \exp\left[(i-1)\sqrt{\frac{\sigma\omega}{2}}\,r\right]\left(\frac{1-i}{\sqrt{2\sigma\omega}\,r} + \frac{1}{\sigma\omega r^2}\right)\sin\vartheta \cdot \sin\varphi. \end{gathered} \tag{2.11}$$

Substituting (2.11) in (2.3), we obtain a system of homogeneous algebraic equations. In the general case the condition of compatibility of these equations leads to clumsy expressions. In the particular case as $\varkappa \to \infty$ the system is compatible if $2k^4 = 815$. This relation, together with (2.10), (2.5), gives the boundary of oscillatory instability Ra and the frequency ω.

§3. In order to study the effect of the terms proportional to the drift velocity u in (1.3), we will consider the following model problem. Let there be a liquid-filled vertical slit in the solid. The dimensions of the slit in the directions of the x and z axes are infinitely large. Lateral heating produces convective motion in the cavity. As a result of the solution effects described above the boundaries of the cavity should drift in the direction of the temperature gradient at infinity A (A is directed along the y axis of the Cartesian coordinate system; see Sec. 1). We denote the drift velocity of the left (colder) boundary by u_- and that of the right (hotter) boundary by u_+. At t = 0 let the width of the cavity be equal to $2a_0$; then at time t the width will be equal to $2a = 2a_0 - u_-t + u_+t$. The center of the cavity (y = 0) migrates at the velocity $u = (u_+ + u_-)/2$.

In this problem the distribution of velocities, temperatures, and concentrations is described by convection equations (1.3) (here, as the unit of length it is necessary to take the half-width of the slit a, the other units being the same as in Sec. 1). The boundary conditions take the form

$$\begin{gathered} T_e = y \ \text{ at } \ y = \pm\infty, \\ \mathbf{v} = 0,\ T = T_e,\ C = KT, \\ \partial T/\partial y - \varkappa \partial T_e/\partial y = \gamma u_\pm,\ \partial C/\partial y = \rho u_\pm \ \text{ at } \ y = \pm 1. \end{gathered} \tag{3.1}$$

Problem (1.3), (3.1) has the exact solution ($\partial/\partial t = 0$)

$$\begin{gathered} T = c_1 + c_2 y + (1/2)\mathrm{Gr}\,\mathrm{Pr}\,uy^2, \\ C = c_3 + c_4 y + (1/2)\mathrm{Gr}\,\mathrm{Sc}\,Kuy^2, \\ T_e(y > 0) = y + c_5 \exp(-\sigma \mathrm{Gr}\,uy) + c_6,\ T_e(y < 0) = y, \\ v_z = (c_1 - c_3)\frac{y^2}{2} + (c_2 - c_4)\frac{y^3}{6} + \frac{\mathrm{Gr}\,u}{24}(\mathrm{Pr} - K\,\mathrm{Sc})\,y^4 + c_7 + c_8 y. \end{gathered}$$

The constants of integration are determined from (3.1). We will write down some of them:

$$c_1 = -(1 + \mathrm{Gr}\,\mathrm{Pr}\,u/2) + (\gamma u_- + \varkappa + \mathrm{Gr}\,u\,\mathrm{Pr}),\ c_2 = \gamma u_- + \varkappa + \mathrm{Gr}u\,\mathrm{Pr},$$

$$u_\pm = \varkappa \frac{\left(1 \pm \frac{K\,\mathrm{Sc}\,\mathrm{Gr}}{\rho}\right)}{\left(\frac{\rho}{K} - \gamma - \mathrm{Gr}\,\mathrm{Pr} + \frac{\gamma K}{\rho}\,\mathrm{Gr}\,\mathrm{Sc}\right)}.$$

The drift velocity

$$u = \varkappa \Big/ \left(\frac{\rho}{K} - \gamma - \mathrm{Gr}\,\mathrm{Pr} + \frac{\gamma K}{\rho}\,\mathrm{Gr}\,\mathrm{Sc}\right).$$

Clearly, the right (hotter) wall moves faster than the left: The cavity grows wider with time. However, the rate of this process is $K\mathrm{Sc}\,\mathrm{Gr}/\rho$ times less than the drift rate u.

In a closed volume, as a result of the incompressibility of the liquid, these effects will generally cause stresses in the crystal; the cavity will not expand.

We will now compare the results obtained with the experimental data. It follows from (1.6), (2.1) that at small Gr the drift of the liquid inclusions is determined by diffusion within the inclusion and that the drift velocity does not depend on the dimensions of the cavity. In the case of lateral heating the correction to the drift velocity $\mathbf{u}_1$ is directed upward and, hence, has practically no effect on the velocity in the direction of the temperature gradient A. Accordingly, in the experiments [1] it is observed that the drift velocity does not depend on the orientation of the temperature gradient at infinity.

At small Gr the convective effects change the direction of the drift velocity: The tangent of the angle between u and A is equal to $\mathrm{Gr}\,|\mathbf{u}_1|/|\mathbf{u}_0|$, where $\mathbf{u}_0$ and $\mathbf{u}_1$ have been written out in (1.6). The effect of convection on mass transfer and the drift of the inclusions has not been studied in this form. The drift velocity calculated from (1.6) and the data of [2] on the properties of an aqueous solution of potassium chloride is equal to $1.1 \cdot 10^{-6}$ cm/sec (for a spherical inclusion in the field of a temperature gradient of 22 deg/cm), which is within the experimental bracket [2].

Our critical Rayleigh number corresponds to instability of diffusion mass transfer for a vertically heated spherical inclusion. On the basis of the data of [2] for an aqueous KCl solution, $\mathrm{Ra}_* = -2$, which corresponds to heating from above. No experiments of this kind have been conducted.

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INVESTIGATION OF THE CONNECTION BETWEEN SOIL AND GROUND WATERS WITH IRRIGATION

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UDC 532.546

With a close occurrence of the level of the ground waters, capillary influx from below can be a significant source of replenishment of the moisture reserves of the root-inhabited layer. The value of this influx depends on the depth of the occurrence of the level of the ground waters, the water and physical properties of the soils of the aeration zone, the form of agricultural cultivations, and the meteorological conditions.

Determination of the rate of the capillary influx is required, in the first place, for calculation of irrigation norms and the irrigation curve and, in the second place, to find the optimal depth of the occurrence of the ground waters, with the aim of preventing processes of secondary salinization, occurring in the case of mineralized ground waters and saline soil waters of the aeration zone [1].

§1. We consider one-dimensional not-fully-established filtration in a vertical direction in a thickness of soil (taking account of its inhomogeneous lithological makeup) from the surface of the ground to the level of the ground waters.

From the solution of the differential equation describing the motion of the water in the unsaturated zone

$$\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial z}\left[k(\theta,z)\left(\frac{\partial\psi}{\partial z}-1\right)\right]-f(\theta,z,t) \tag{1.1}$$

with the initial and boundary conditions

$$\psi(z,\ 0)=\psi^0(z),\ 0\leqslant z\leqslant l,\ t=0; \tag{1.2}$$

$$-k(\partial\psi/\partial z-1)=R(t),\ z=0,\ t>0; \tag{1.3}$$

$$\psi=0,\ z=l,\ t\geqslant 0 \tag{1.4}$$

and conditions of conjugation in the form of the equality of the pressures at the interface between the layers, a determination is made of the pressure $\psi(z, t)$ and, consequently, of the moisture content $\theta(z, t)$, if, for each lithological layer of the soil thickness under investigation, we know the main hydrophysical characteristics $\theta(\psi)$ and $k(\theta)$ [or $k(\psi)$], which are here assumed to be single-valued.

The function $f(\theta, z, t)$ in Eq. (1.1) takes account of the absorption of moisture by the roots of plants in the region $0 \le z \le z_r(t)$, where $z_r(t)$ is the thickness of the root zone. For $z > z_r$, we assume that $f \equiv 0$.

The rate of capillary influx from the ground waters into the aeration zone v is found from the balance equation

$$v(t)=\frac{dw}{dt}-R+\int_0^{z_r} f dz, \tag{1.5}$$

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